# MODELLING AND SIMULATION CONCEPTS

Modern science would be inconceivable without computers to gather data and run model simulations. Whether it involves bringing back pictures of the surface of the planet Mars or detailed images to guide brain surgeons, computers have greatly extended our knowledge of the world around us and our ability to turn ideas into engineering reality. Thus modelling and computer simulation are important interdisciplinary tools.

# Definitions

1. **Modelling** is the process of generating abstract, conceptual, graphical and/or mathematical models. Science offers a growing collection of methods, techniques and theory about all kinds of specialized scientific modelling.

**Modelling** also means to find relations between systems and models. Stated otherwise, models are abstractions of real or imaginary worlds we create to understand their behaviour, play with them by performing "what if" experiments, make projections, animate or simply have fun.

1. A **model** in general is a pattern, plan, representation (especially in miniature), or description designed to show the main object or workings of an object, system, or concept.
2. A **model** (physical or hypothetical) is a representation of real-world phenomenon or elements (objects, concepts or events). Stated otherwise a model is an attempt to express a *possible structure of physical causality*.

Models in science are often theoretical constructs that represent any particular thing with a set of variables and a set of logical and or quantitative relationships between them. Models in this sense are constructed to ensure reasoning within an idealized logical framework about these processes and are an important component of scientific theories.

1. **Simulation -**is the manipulation of a model in such a way that it operates on time or space to compress it, thus enabling one to perceive the interactions that would not otherwise be apparent because of their separation in time or space.



1. Modelling and Simulation is a discipline for developing a level of

understanding of the interaction of the parts of a system, and of the system as a whole. The level of understanding which may be developed via this discipline is seldom achieved via

any other discipline.

1. A **computer model** is a simulation or model of a situation in the real world or an imaginary world which has parameters that the user can alter.

For example Newton considers movement (of planets and of masses) and writes

equations, among which *f* = *ma* (where f is force, m mass and a acceleration), that make the dynamics intelligible. Newton by this expression makes a formidable proposition, that force causes acceleration, with mass as proportionality coefficient.

# What is Modelling and Simulation?

**Simulation**, as used in decision making, is a descriptive technique in which a model of a process is developed and then experiments are conducted on the model to evaluate its behavior under various conditions. Simulation is not an optimizing technique. It does not produce a solution per se. Instead, simulation ensures decision makers to test their solutions on a model that reasonably duplicates a real process. Simulation models ensure decision makers to experiment with decision alternatives using *what-if* approach.

**Modelling** is a discipline for developing a level of understanding of the interaction of the parts of a system, and of the system as a whole. The level of understanding which may be developed via this discipline is seldom achieved via any other discipline.

A simulation is a technique (not a method) for representing a dynamic real world system by a model and experimenting with the model in order to gain information about the system and therefore take appropriate decision. Simulation can be done by hand or by a computer. Simulations are generally iterative in their development. One develops a model, simulates it, learns from the result, revises

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the model, and continues the iterations until an adequate level of understanding is

attained.

Modelling and Simulation is a discipline, it is also very much an art form. One can learn about riding a bicycle from reading a book. To really learn to ride a bicycle one must become actively engaged with a bicycle. Modelling and Simulation follows much the same reality. You can learn much about modelling and simulation from reading books and talking with other people. Skill and talent in developing models and performing simulations is only developed Through the building of models and simulating them. It is very much “learn as you go” process. From the interaction of the developer and the models emerges an understanding of what makes sense and what doesn't.

# Type of Models

There are many types of models and different ways of classifying/grouping them. For simplicity, Models may be grouped into the following – Physical, Mathematical, Analogue, Simulation, Heuristic, Stochastic and Deterministic models.

1. Physical Models

These are call iconic models. Good examples of physical models are model cars, model railway, model airplane, scale models, etc. A model railway can be used to study the behaviour of a real railway, also scale models can be used to study a plant layout design. In simulation studies, iconic models are rarely used.

1. Mathematical Models

These are models used for predictive (projecting) purposes. They are abstract and take the form of mathematical expressions of relationships. For examples:

|  |  |  |
| --- | --- | --- |
| 1. | x2 + y2 = | 1(mathematical model of a circle of radius 1) |
|  |  | Principal x Rate x Time |
| 2. | Interest | =…………………………… |
|  |  | 100 |

3. Linear programming models and so on.

Mathematical models can be as simple as interest earnings on a savings account or as complex as the operation of an entire factory or landing astronauts on the

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moon. The development of mathematical models requires great deal of skill and

knowledge.

1. Analogue Models

These are similar to iconic models. But here some other entities are used to represent directly the entities of the real world. An example is the analogue computer where the magnitudes of the electrical currents flowing in a circuit can be used to represent quantities of materials or people moving around in a system. Other examples are; the gauge used to check the pressure in a tyre. The movement of the dial represent the air pressure in the tyre.

In medical examination, the marks of electrical current on paper, is the analogue representation of the working of muscles or organs.

1. Simulation Models

Here, instead of entities being represented physically, they are represented by sequences of random numbers subject to the assumptions of the model. These models represent (emulate) the behaviour of a real system. They are used where there are no suitable mathematical models or where the mathematical model is too complex or where it is not possible to experiment upon a working system without causing serious disruption.

1. Heuristic Models

These models use intuitive (or futuristic) rules with the hope that it will produce worked solutions, which can be improved upon. For example, the Arthur C Clerk’s heuristic model was the forerunner of the communications satellite and today’s international television broadcast.

1. Deterministic Models

These are models that contain certain known and fixed constants throughout their formulation e.g., Economic Order Quantity (EOQ) for inventory control under uncertainty.

1. Stochastic models

These are models that involve one or more uncertain variable and as such

are subject to probabilities.

# Advantages of Using Models

1. They are safer.
2. They are less expensive. For example, Practical Simulators are used to train pilots.
3. They are easier to control than the real world counterparts.

# Applications

One application of scientific modelling is the field of “Modelling and Simulation", generally referred to as "M&S". M&S has a spectrum of applications which range from concept development and analysis, through experimentation, measurement and verification, to disposal analysis. Projects and programs may use hundreds of different simulations, simulators and model analysis tools

# Modelling Procedure

In modelling we construct a suitable representation of an identified real world problem, obtain solution(s) for that representation and interpret each solution in terms of the real situation. The steps involved in modelling are as follows:

1. Examine the real world situation.
2. Extract the essential features from the real world situation.
3. Construct a model of the real (object or system) using just the essential features identified.
4. Solve and experiment with the model.
5. Draw conclusions about the model.
6. If a further refinement necessary, then re-examine the model and readjust parameters and continue at 4, otherwise continue at 7.
7. Proceed with implementation.

Explanation of the Steps

Begin with the real world situation, which is to be investigated with a view to solving some problem or improving that situation.

The first important step is to extract from the real world situation the essential features to be included in the model. Include only factors that make the model a meaningful representation of reality, while not creating a model, which is difficult by including many variables that do not have much effect. Factors to be considered include ones over which management has control and external factors beyond management control. For the factors included, assumptions have to be made about their behaviour.

Run (simulate) the model and measure what happens. For example, if we have simulation of a queuing situation where two servers are employed, we can run this for hundreds of customers passing through the system and obtain results such as the average length of the queue and the average waiting time per customer. We can then run it with three servers, say, and see what new values are obtained for these parameters. Many such runs can be carried out making different changes to the structure and assumptions of the model.

In the case of a mathematical model we have to solve a set of equations of some sort, e.g. linear programming problem where we have to solve a set of constraints as simultaneous equations, or in stock control – where we have to use previously accumulated data to predict the future value of a particular variable.

When we have solved our mathematical model or evaluated some simulation runs, we can now draw some conclusions about the model. For example, if we have the average queue length and the average waiting time for a queuing situation varied in some ways, we can use this in conjunction with information on such matters as the wage-rates for servers and value of time lost in the queue to arrive at decisions on the best way to service the queue.

Finally, we use our conclusions about the model to draw some conclusions about the original real world situation. The validity of the conclusions will depend on how well our model actually represented the real world situation.

Usually the first attempt at modelling the situation will almost certainly lead to results at variance with reality. We have to look back at the assumptions in the model and adjust them. The model must be rebuilt and new results obtained. Usually, a large number of iterations of this form will be required before

acceptable model is obtained. When an acceptable model has been obtained, it is

necessary to test the sensitivity of that model to possible changes in condition.

The modelling process can then be considered for implementation when it is decided that the model is presenting the real world (object or system) sufficiently well for conclusions drawn from it to be a useful guide to action.

The model can be solved by hand, especially if it is simple. It could take time to arrive at an acceptable model. For complex models or models which involve tremendous amount of data, the computer is very useful.

# Exercises

Differentiate between Model, Modelling, Simulation and Computer model. What are the steps followed in modelling?

# SIMULATION MODELING STEPS

The application of simulation involves specific steps in order for the simulation study to be successful. Regardless of the type of problem and the objective of the study, the process by which the simulation is performed remains constant. A simulation of a system is the operation of a model of the system; “Simulation Model”. The steps involved in developing a simulation model, designing a simulation experiment, and performing simulation analysis are:

1. **Problem Definition -** The initial step involves defining the goals of the study and determining what needs to be solved. The problem is further defined through objective observations of the process to be studied. Care should be taken to determine if simulation is the appropriate tool for the problem under investigation.
2. **Project Planning -** The tasks for completing the project are broken down into work packages with a responsible party assigned to each package. Milestones are indicated for tracking progress. This schedule is necessary to determine if sufficient time and resources are available for completion.
3. **System Definition -** This step involves identifying the system components to be modeled and the performance measures to be analyzed. Often the system is very complex, thus defining the system requires an experienced simulator who can find the appropriate level of detail and flexibility.
4. **Model Formulation -** Understanding how the actual system behaves and determining the basic requirements of the model are necessary in developing the right model. Creating a flow chart of how the system operates facilitates the understanding of what variables are involved and how these variables interact.
5. **Input Data Collection & Analysis -** After formulating the model, the type of data to collect is determined. New data is collected and/or existing data is gathered. Data is fitted to theoretical distributions. For example, the arrival rate of a specific part to the manufacturing plant may follow a normal distribution curve.
6. **Model Translation -** The model is translated into programming language. Choices range from general purpose languages such as Fortran or simulation programs such as Arena.
7. **Verification & Validation -** Verification is the process of ensuring that the model behaves as intended, usually by debugging or through animation. Verification is necessary but not sufficient for validation, that is a model may be verified but not valid. Validation ensures that no significant difference exists between the model and the real system and that the model reflects reality. Validation can be achieved through statistical analysis. Additionally, face validity may be obtained by having the model reviewed and supported by an expert.
8. **Experimentation & Analysis -** Experimentation involves developing the alternative model(s), executing the simulation runs, and statistically comparing the alternative(s) system performance with that of the real system.
9. **Documentation & Implementation -** Documentation consists of the written report and/or presentation. The results and implications of the study are discussed. The best course of action is identified, recommended, and justified.

## Decisions for Simulating

Completing the required steps of a simulation study establishes the likelihood of the study's success. Although knowing the basic steps in the simulation study is important, it is equally important to realize that not every problem should be solved using simulation. In the past, simulation required the specialized training of programmers and analysts dedicated to very large and complex projects. Now, due to the large number of software available, simulation at times is used inappropriately by individuals lacking the sufficient training and experience. When simulation is applied inappropriately, the study will not produce meaningful results. The failure to achieve the desired goals of the simulation study may induce blaming the simulation approach itself when in fact the cause of the failure lies in the inappropriate application of simulation [8].

To recognize if simulation is the correct approach to solving a particular problem, four items should be evaluated before deciding to conduct the study:

Type of Problem

* 1. Availability of Resources
  2. Costs
  3. Availability of Data

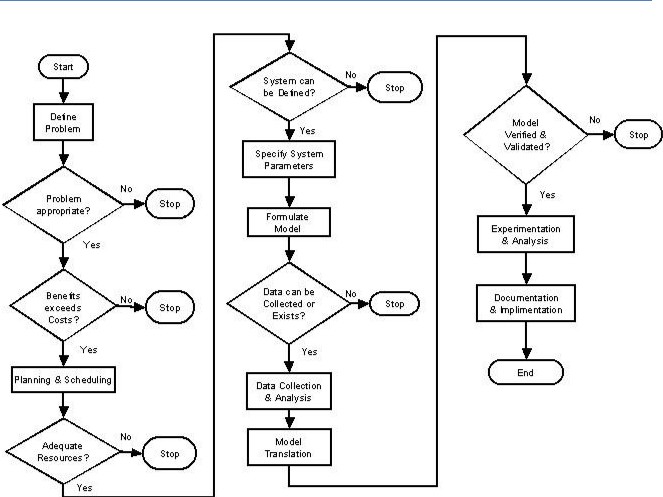
1. ***Type of Problem*:** If a problem can be solved by common sense or analytically,

the use of simulation is unnecessary. Additionally, using algorithms and mathematical equations may be faster and less expensive than simulating. Also, if the problem can be solved by performing direct experiments on the system to be evaluated, then conducting direct experiments may be more desirable than simulating. To illustrate, recently the UH Transportation Department conducted field studies on expanding the campus shuttle system. The department used their own personnel and vehicles to perform the experiment during the weekend. In contrast, developing the simulation model for the shuttle system took one student several weeks to complete. However, one factor to consider when performing directing experiments is the degree in which the real system will be disturbed. If a high degree of disruption to the real system will occur, then another approach may be necessary. The real system itself plays another factor in deciding to simulate. If the system is too complex, cannot be defined, and not understandable then simulation will not produce meaningful results. This situation often occurs when human behavior is involved.

1. ***Availability of Resources*:** People and time are the determining resources for conducting a simulation study. An experienced analyst is the most important resource since such a person has the ability and experience to determine both the model's appropriate level of detail and how to verify and validate the model. Without a trained simulator, the wrong model may be developed which produces unreliable results. Additionally, the allocation of time should not be so limited so as to force the simulator to take shortcuts in designing the model. The schedule should allow enough time for the implementation of any necessary changes and for verification and validation to take place if the results are to be meaningful.
2. ***Costs*:** Cost considerations should be given for each step in the simulation process, purchasing simulation software if not already available, and computer resources. Obviously if these costs Exceed the potential savings in altering the current system, then simulation should not be pursued.
3. ***Availability of Data*:** The necessary data should be identified and located, and if the data does not exist, then the data should be collectible. If the data does not exist and cannot be collected, then continuing with the simulation study will eventually yield unreliable and useless results. The simulation output cannot be compared to the real system's performance, which is vital for verifying and validating the model.

The basic steps and decisions for a simulation study are incorporated into a flowchart as shown below:

# Steps and Decisions for Conducting a Simulation Study



**Figure 1: Steps and Decisions for Conducting a Simulation Study**



Once simulation has been identified as the preferred approach to solving a particular problem, the decision to implement the course of action suggested by the simulation study's results does not necessarily signify the end of the study, as indicated in the flowchart above. The model may be maintained to check the system's response to variability experienced by the real system. However, the

extent to which the model may be maintained largely depends on the model's

flexibility and what questions the model was originally designed to address.

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1. **RANDOM NUMBERS**

# Introduction

The use of Random numbers lies at the foundation of modeling and simulations. Computer applications such as simulations, games, graphics, etc., often need the ability to generate random numbers for such application.

The quality of a random number generator is proportional to its **period**, or the number of random numbers it can produce before a repeating pattern sets in. In large-scale simulations, different algorithms (called shift-register and lagged- Fibonacci) can be used, although these also have some drawbacks, combining two different types of generators may produce the best results.

# How to generate random numbers

**Random Number** can be defined as numbers that show no consistent pattern, with each number in a series and are neither affected in any way by the preceding number, nor predictable from it.

One way to get random digits is to simply start with an arbitrary number with a specified number of digits, for example 4 digits. The first number is called the **seed**. The seed is multiplied by a **constant** number of the same number of digits (length), and the desired number of digits is taken off the right end of the product. The result becomes the new seed.

It is again multiplied by the original constant to generate a new product, and the process is repeated as often as desired. The result is a series of digits that appear randomly distributed as though generated by throwing a die or spinning a wheel. This type of algorithm is called a

# congruential generator

The so-called true random number generators extract random numbers from physical phenomena such as a radioactive source or even atmospheric noise as detected by a radio receiver.

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* 1. **Pseudorandom Number Generation**

In this section we look at how random numbers may be generated by human beings for use in simulating a system or by computer for use while simulating an event. What we usually do is to take for instance ten pieces of papers and number them 0,1,2,3,4,5,6,7,8, and 9 , fold and place them in a box. Shake the box and thoroughly mix the slips of paper. Select a slip; then record the number that is on it. Replace the slip and repeat this procedure over and over.

The resultant record of digits is a realized sequence of random numbers. Assuming you thoroughly mix the slips before every draw, the nth digit of the sequence has an equal or uniform chance of being any of the digits 0,1,2,3,4,5,6,7,8, and 9 irrespective of all the preceding digits in the recorded sequence.

# Random Numbers in Computer

*How does computer generate a sequence of random numbers?*

One way is to perform the above “slip-in-a-box” experiment and then store the recorded sequence in a computer-backing store. The RAND Corporation using specially designed electronic equipment, to perform the experiment, actually did generate a table of a million random digits. The table can be obtained on tape, so that blocks of the numbers can be read into the memory of a high- speed computer, as they are needed. Their approach is disadvantageous since considerable computer time was expended in the delays of reading numbers into memory from a tape drive.

Experts in computer science have devised mathematical processes for generating digits that yield sequences satisfying many of the statistical properties of a truly random process. Since such a process is not really random, it is called **pseudo-random number generator**.

# Using the RAND Function in Excel

The Microsoft Excel has a numeric function named RAND, which generates random numbers between 0 and 1. Each time RAND is executed, a pseudo random number between 0 and 1 is generated. RAND returns an evenly distributed random real number greater than or equal to

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0 and less than 1. A new random real number is returned every time the

worksheet is calculated. RAND uses the following syntax**:** RAND (). The RAND function syntax has no arguments.

Using RAND function at any time will always generate the same sequence

Of pseudo random numbers unless we vary the random number seed using the Excel statement: To generate a random real number between a and b, use: RAND ()\*(b-a)+a If you want to use RAND to generate a random number but don't want the numbers to change every time the cell is calculated, you can enter =RAND() in the formula bar, and then press F9 to change the formula to a random number.

# Example

**Table 1: Formula and Description**

|  |  |  |
| --- | --- | --- |
| **Formula** | **Description** | **Resul t** |
|  | | |
| =RAND() | A random number greater than or equal to 0 and less than 1 (varies) | varie s |
|  | | |
| =RAND()\*1 00 | A random number greater than or equal to 0 but less than 100 (varies) | varie s |

# Simulating Randomness

Suppose we want to simulate the throwing of a fair die. A random number between 0 and1 will not always satisfy our needs. If the die is fair, throwing it several times will yield a series of uniformly distributed integers 1, 2,3,4,5 and 6. Consequently we need to be Able to generate a random integer with values in the range 1 and 6 inclusive.

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Now the function RAND generates a random number between 0 and 1.

Specifically, the random variable X is in the range: 0 < X < 1. The expression X= RAND ()\*6. Will generate a number in the range: 0 < X < 6.

* 1. **Properties of a Good Random Number Generator** The random numbers generated should;
     1. Have as nearly as possible a uniform distribution
     2. Should be fast
     3. Not require large amounts of memory
     4. Have a long period
     5. Be Able to generate a different set of random numbers or a series of numbers
     6. Not degenerate.

1. **METHODS OF GENERATING RANDOM NUMBERS** The following are methods of generating random numbers:
   1. Congruential Method
   2. Quadratic Congruential Method
   3. Mid-square method
   4. Mid-product Method
   5. Fibonnachi Method

# The Congruential Method

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A **linear congruential generator** (**LCG**) is an algorithm that yields a sequence of

pseudo-randomized numbers calculated with a discontinuous piecewise linear equation. The method represents one of the oldest and best-known pseudorandom number generator algorithms. The theory behind them is relatively easy to understand, and they are easily implemented and fast, especially on computer hardware which can provide modulo arithmetic by storage-bit truncation.

The generator is defined by the recurrence relation:



where is the sequence of pseudorandom values, and



* the "modulus"



* + the "multiplier"



* + the "increment"



– the "seed" or "start value"

# Example 1

For example, the sequence obtained when X0 = a = c = 7, m = 10, is 7, 6, 9, 0, 7, 6, 9, 0, ...

# The Quadratic congruential method

This method uses the formula:

Xn=1 = (dX 2+ cXn + a) nmodulo m

Where d is chosen in the same way as c and m should be a power of 2 for the

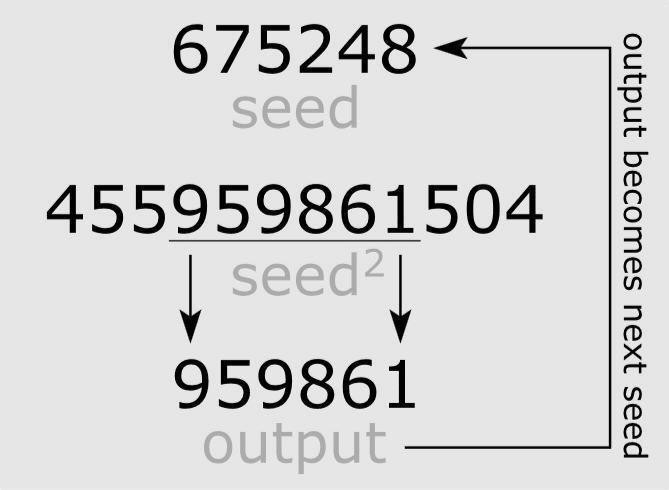
method to yield satisfactory results.

# The Mid-square method

The middle square method consists of taking the square of the previous random number and to extract the middle digits. This method gives rather poor results since generally sequences tend to get into a short periodic orbit.

The first random number is generated from the seed by squaring the seed and discarding all the digits except the middle four digits. This number is subsequently used as the new seed to generate the next random number in the same manner and so on.

The formula is: Xn+1 =nX 2

The mid-square method is rarely used these days as it has the tendency to degenerate rapidly. Also, if the number zero is ever generated, then all subsequent numbers generated will be zero. Furthermore, the method is slow when simulated in the computer since many multiplications and divisions are required to access the middle four digits.

# Example

If we generate 4-digit numbers starting from 3567 we obtain 7234 as the next number since the

square of 3567 equals 12723489. Continuing in the same way the next number will be 3307.

The resulting sequence enters already after 46 iterations a periodic orbit:

# The mid-product method

This method is similar to the mid-square method, except that a successive random number is obtained by multiplying the current number by a constant c, and taking the middle digits.

The formula is: Xn+1 = cXn

The mid-product method has a longer period and it is more uniformly distributed than the mid-square method.

# The Fibonacci method

Fibonacci method uses the formula: Xn+1 = (Xn + Xn-1)

modulo m

In this method, two initial seeds need to be provided. However, experience has shown that the random numbers generated by using Fibonacci method fail to pass tests for randomness. Therefore, the method does not give satisfactory results.

From the foregoing discussions, it is obvious that the last three methods – mid-square, mid-product and Fibonacci are of historical significance and have detrimental and limiting characteristics.

# Exercises

* + 1. Write a Excel program using Quadratic congruential method to generate 15 random integer numbers between 1 and 50.
    2. Produce at Able of random numbers using multiplicative congruential method, using a =5, m =8 and X0 = 4. Draw an inference from your solution.
    3. Write a Excel function that can be referenced as computer random number between 30 and 100 using mixed congruential method.
    4. Use the mixed congruential method to generate the following sequences of random numbers:

a. A sequence of 10 one-digit random numbers given that

Xn+1 (Xn + 3)(modulo 10) and X0 = 2

1. A sequence of eight random numbers between 0 and 7 given that

Xn+1 (5Xn + 1)(modulo 8) and X0 = 4

1. A sequence of two-digit random numbers such that

Xn+1 (61Xn + 27)(modulo 100) and X0 = 40

1. A sequence of five-digit random numbers such that

Xn+1 (21Xn + 53)(modulo 100) and X0 = 33

=

u r x 10-1

n n

= (modulo 1010), and r = any odd number not divisible by 5,

where r 100003r n- then the

n 1 0

period of the sequence will be 5 x 108, that is r

= for the first time at n = 5 x

r 108 and the

n 0

cycle subsequently repeat itself. As an example, using our mixed congruential formula

Xn+1 = (aXn+c) (modulo m),

And suppose m = 8, a = 5, c = 7 and X0 (seed) = 4 we can generate a random sequence of integer numbers thus:

# Table 2: Random sequence of integer numbers

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| n | Xn+1 = (5Xn+7)mod  8 | | | | | | | |
| 0 | X 1 | = | (5\*X0+7)mod 8  = | | (5\*X4+7)mod 8  = | | 27 mod 8 = 3 | |
| 1 | X 2 | = | (5\*X1+7)mod 8  = | | (5\*X3+7)mod 8  = | | 22 mod 8 = 6 | |
| 2 | X 3 | = | (5\*X2+7)mod 8  = | | (5\*X6+7)mod 8  = | | 37 mod 8 = 5 | |
| 3 | X 4 | = | (5\*X3+7)mod 8  = | | (5\*X5+7)mod 8  = | | 32 mod 8 = 0 | |
| 4 | X 5 | = | (5\*X4+7)mod 8  = | | (5\*X0+7)mo d 8 | = | 7 mod  8 | = 7 |
| 5 | X 6 | = | (5\*X5+7)mod 8 | = | (5\*X7+7)mo d 8 | = | 42 mod  8 | = 2 |
| 6 | X 7 | = | (5\*X6+7)mod 8 | = | (5\*X2+7)mo d 8 | = | 17 mod  8 | = 1 |
| 7 | X 8 | = | (5\*X7+7)mod 8 | = | (5\*X1+7)mo d 8 | = | 12 mod  8 | = 4 |

Note that the value of X8 is 4, which is the value of the seed X0. So if we compute X9, X10, etc the same random numbers 3, 6,5,0,7,2,1,4 will be generated once more. Note also that if we divide the random integer values by 8, we obtain random numbers in the range 0 < Xn+1 < 1 which is similar to using the RAND function of Excel

**Monte Carlo simulation**

Monte Carlo simulation is a technique used to study how a model responds to randomly generated inputs. It typically involves a three-step process:

1. Randomly generate “N” inputs (sometimes called scenarios).
2. Run a simulation for each of the “N” inputs. Simulations are run on a computerized model of the system being analyzed.
3. Aggregate and assess the outputs from the simulations. Common measures include the mean value of an output, the distribution of output values, and the minimum or maximum output value.

Systems analyzed using Monte Carlo simulation include financial, physical, and mathematical models. Because simulations are independent from each other, Monte Carlo simulation lends itself well to parallel computing techniques, which can significantly reduce the time it takes to perform the computation.

# Example 1

The manager of a machine shop is concerned about machine breakdowns. He has made a decision to simulate breakdowns for a 10-day period. Historical data on breakdowns over the last 100 days are given in the following:

# Table 3: Historical data on breakdowns over the last 100 days

|  |  |
| --- | --- |
| **Number of Breakdowns** | **Frequency** |
| 0 | 10 |
| 1 | 30 |
| 2 | 25 |
| 3 | 20 |
| 4 | 10 |
| 5 | 5 |
| **Total** | 100 |

1. Develop a cumulative frequency probability and assign random number intervals corresponding to it
2. Simulate breakdowns for a 10-day period.
3. What is the mean breakdown for the 10-day period
4. Compare simulated mean to the *expected* number of breakdowns based on the historical data

**Note:** The random number are obtained from the table as 18 25 73 12 54 96 23 31

45 01

# Solution

Develop cumulative probabilities for breakdowns:

* 1. Convert frequencies into relative frequencies by dividing each frequency by the sum of the frequencies. Thus, 10 becomes 10/100, 10, 30 becomes 30/100, 30, and so on.
  2. Develop cumulative relative frequencies (i.e., cumulative probabilities) by successive summing. The results are shown in the following table:

# Table 4: Cumulative probabilities for breakdowns

|  |  |  |  |
| --- | --- | --- | --- |
| **No. of Breakdowns** | **Frequency** | **Relative Frequency** | **Cumulative Frequency** |
| 0 | 10 | 0.10 | 0.10 |
| 1 | 30 | 0.30 | 0.40 |
| 2 | 25 | 0.25 | 0.65 |
| 3 | 20 | 0.20 | 0.85 |
| 4 | 10 | 0.10 | 0.95 |
| 5 | 5 | 0.05 | 1.00 |
| Total | 100 | 1.00 |  |

**b.** Assign random-number intervals to correspond to the cumulative probabilities for breakdowns. (Note: Use two-digit numbers because the cumulative probabilities are given to two decimal places.). You want a 10 percent probability of obtaining the event “0 breakdowns” in our simulation. Therefore, you must designate 10 percent of the possible random numbers as corresponding to that event. There are 100 two digit numbers, so we can assign the 10 numbers 01 to 10 to that event. Similarly, assign the numbers 11 to 40 to “one breakdown,” 41 to 65 to “two breakdowns,” 66 to 85 to “three breakdowns,” 86 to 95 to “4 breakdowns” and 96 to 00 to “five breakdowns.” Note that “00” is assigned as if it was “100,” not “0.” This makes the interval maximums correlate well to the cumulative Probabilities.

# Table 5:Interval maximums correlations with cumulative Probabilities

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **No. of** | **Frequency** | **Relative** | **Cumulative** | **Correspondin g** | |
| **Breakdowns** | **Frequency** | **Frequency** | **Random No.** | |
| 0 | 10 | 0.1  0 | 0.10 | 0  1 | – 10 |
| 1 | 30 | 0.3  0 | 0.40 | 1  1 | – 40 |
| 2 | 25 | 0.2  5 | 0.65 | 4  1 | – 65 |
| 3 | 20 | 0.2 | 0.85 | 6 | – 85 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | |  |
|  |  | 0 |  | 6 | |  |
| 4 | 10 | 0.1  0 | 0.95 | 8  6 | – 95 |  |
| 5 | 5 | 0.0  5 | 1.00 | 9  6 | - 100 |  |
| Total | 100 | 1.0  0 |  |  | | |

**d.** Convert the random numbers into numbers of breakdowns on each day, starting from day 1: 18 falls in the interval 11 to 40 and corresponds, therefore, to one breakdown on day 1. While 25 falls in the interval 11 to 40 and corresponds to one breakdown on day 2, and so on repeat for each day. The table below summarizes the simulation.

# Table 6: Conversion of random numbers into numbers of breakdowns

|  |  |  |
| --- | --- | --- |
| **Day** | **Random No.** | **Simulated No. of Breakdown** |
| 1 | 18 | 1 |
| 2 | 25 | 1 |
| 3 | 73 | 3 |
| 4 | 12 | 1 |
| 5 | 54 | 2 |
| 6 | 96 | 5 |
| 7 | 23 | 1 |
| 8 | 31 | 1 |
| 9 | 45 | 2 |
| 10 | 01 | 0 |
| Total |  | 17 |

a) The mean number of breakdowns for this 10-day simulation is 17/10 = 1.7 breakdowns per day.

b) The historical data: 0(.10) + 1(.30) + 2(.25) + 3(.20) + 4(.10) + 5(.05) =

2.05 per day.

c) The two (1.7 and 2.05) are close but because of random variability, not

equal. As the length of simulation increases, however, the sample mean should approach the population mean.

# Exercise 1

The average number of lost-time accidents at Bamburi Cement Factory has been determined from historical records to be 2 accidents per day. Moreover, it has been determined that this accident rate can be well approximated by a Poisson distribution. Historical data on breakdowns over the last 100 days are given in the following table:

# Table 7: Average number of lost-time accidents at Bamburi Cement Factory

|  |  |
| --- | --- |
| **Number of Accidents** | **Frequency** |
| 0 | 135 |
| 1 | 271 |
| 2 | 271 |
| 3 | 180 |
| 4 | 90 |
| 5 | 36 |
| **6** | 12 |
| **7** | 4 |
| **8** | 1 |
| **Total** | 1000 |

1. Develop a cumulative frequency probability and assign random number intervals corresponding to it
2. Simulate 5-days of accident experience for the factory
3. What is the mean breakdown for the 5-day period
4. Compare simulated mean to the *expected* number of breakdowns based on the historical data

**Note:** The random number are obtained from the table as 182 251 735 124 and 549.

# SOLUTION

**Table 8: Random number obtained from the table**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **No. of Accidents** | **Frequency** | **Relative** | **Cumulative** | **Correspondin g** | |
| **Frequency** | **Frequency** | **Random No.** | |
| 0 | 135 | 0.135 | 0.135 | 00  1 | – 135 |
| 1 | 271 | 0.271 | 0.406 | 13  6 | – 406 |
| 2 | 271 | 0.271 | 0.677 | 40  7 | – 677 |
| 3 | 180 | 0.180 | 0.857 | 67  8 | – 857 |
| 4 | 90 | 0.900 | 0.947 | 858– 947 | |
| 5 | 36 | 0.360 | 0.983 | 94  8 | - 983 |
| **6** | 12 | 0.120 | 0.995 | 98  4 | - 995 |
| **7** | 4 | 0.400 | 0.999 | 99  6 | - 999 |
| **8** | 1 | 0.100 | 0.100 | 1000 | |
| **Total** | 1000 | 1.000 |  |  | |

# Table 9: Simulation

|  |  |  |
| --- | --- | --- |
| **Day** | **Random No.** | **Simulated No. of Breakdown** |
| 1 | 182 | 1 |
| 2 | 251 | 1 |
| 3 | 735 | 3 |
| 4 | 124 | 0 |
| 5 | 549 | 2 |
| Total |  | 7 |

* 1. The mean number of breakdowns for this 5-day simulation is 7/5 = 1.4

accidents per day.

* 1. The historical data: 2 accidents per day.
  2. The two (1.4 and 2.0) are close but because of random variability, not equal. As the length of simulation increases, however, the sample mean should approach the population mean.

# Exercise 2

The manager of Car and General Company wants to acquire some insight into how a proposed a policy for reordering Tuktuks from the manufacturer in India might result in shortage. Under the new policy, 2 Tuktuks are to be ordered whenever the number of Tuktuks on hand at the end of the day plus number of Tuktuks on order is two or fewer. Assume that purchase lead time is only 2 full days and beginning stock is 4 Tuktuks. According to the dealer’s records, the probability distribution for daily demand (i.e., its sales) is:

# Table 10: Probability distribution for daily demand

|  |  |
| --- | --- |
| **Tuktuk Demand** | **Probability of Demand** |
| 0 | 0.50 |
| 1 | 0.40 |
| 2 | 0.10 |

1. Develop a cumulative frequency probability and assign random number intervals corresponding to it
2. Manually simulate the inventory system for10 days
3. What is the probability of Tuktuks being out of stock with the new policy?

**Note:** The random number are obtained from the table as 54 73 29 51 87 51 99 18

30 and 27.

**SOLUTION**

**Table 11: Cumulative frequency probability**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Tuktuk** | **Probability of** | **Cumulative Probability** | **Ranges** | |
| **Demand** | **Demand** | **of Demand** |
| 0 | 0.50 | 0.50 | | 01–50 |
| 1 | 0.40 | 0.90 | | 51–90 |
| 2 | 0.10 | 1.00 | | 91 – 100 |

# Table 12: Manual Simulation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Day** | **Beginnin**  **g** | **Random** | **Deman**  **d** | **for** | **Ending** | **On order** |
| **Inventor**  **y** | **No.** | **Tuktu**  **ks** |  | **Inventor**  **y** |
| 1 | 4 | 54 | 1 | | 3 | - |
| 2 | 3 | 73 | 1 | | 2 | Reorder = 2; arrival day = 5 |
| 3 | 2 | 29 | 0 | | 2 | 2 |
| 4 | 2 | 51 | 1 | | 1 | 2 |
| 5 | 1+ (Day 2  order | 87 | 1 | | 2 | Reorder = 2; arrival day = 8 |
| arrived) =  3 |
| 6 | 2 | 51 | 1 | | 1 | 2 |
| 7 | 1 | 99 | 2 | | -1 | Reorder = 2; arrival day = 10 |
| 8 | (Day  5order | 18 | 0 | | 1 | 2 |
| arrived) –  1 = 1 |
| 9 | 1 | 30 | 0 | | 1 | 2 |
| 10 | (Day  7order | 27 | 0 | | 3 |  |
| arrived) +  1 = 3 |

1. The only negative Ending Inventory is in Day 7. Therefore, estimate for probability of shortage = 1/10 = 0.10.

# Exercise 3

The number of orders received by Furniture Rama is to be simulated for 8 days period. The shop manager has collected the following data for the number of orders received daily.

**Table 13: Number of orders received by Furniture Rama**

|  |  |
| --- | --- |
| **No. of Orders** | **Frequency** |
| Less than 2 | 0 |
| 3 | 10 |
| 4 | 50 |
| 5 | 80 |
| 6 | 40 |
| 7 | 16 |
| 8 | 4 |
| Greater than 9 | 0 |

1. Develop a cumulative frequency probability and assign random number intervals corresponding to it
2. Simulate breakdowns for an 8-day period.
3. What is the average number of orders per day for 8-day simulation period
4. Compare simulated average to the *expected* number of orders based on the historical data

**Note:** The random number are obtained from the table as 18 25 73 12 54 96 23

and 31

# SOLUTION

**Table 14: Cumulative frequency probability for number of orders**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **No. of Orders** | **Frequency** | **Relative** | **Cumulative** | **Correspondin g** | | |
| **Frequency** | **Frequency** | **Random No.** | | |
| Less than 2 | 0 | 0.00 | 0.00 | 0.0 | | – 0.0 |
| 3 | 10 | 0.05 | 0.05 | 0.0 | | – 5.0 |
| 4 | 50 | 0.25 | 0.30 | 6.0 | | – 30 |
| 5 | 80 | 0.40 | 0.70 | 3  1 | – 70 | |
| 6 | 40 | 0.20 | 0.90 | 7  1 | – 90 | |
| 7 | 16 | 0.08 | 0.98 | 9  1 | - 98 | |
| 8 | 4 | 0.02 | 1.00 | 9  9 | - 100 | |
| Greater than 9 | 0 | 0.00 | 1.00 | 10  0 | | |
| **Total** | **200** | **1.00** |  |  | | |

# Table 15: Simulation breakdowns for an 8-day period

|  |  |  |
| --- | --- | --- |
| **Day** | **Random No.** | **Simulated No. of Breakdown** |
| 1 | 18 | 4 |
| 2 | 25 | 4 |
| 3 | 73 | 6 |
| 4 | 12 | 4 |
| 5 | 54 | 5 |
| 6 | 96 | 7 |
| 7 | 23 | 4 |
| 8 | 31 | 5 |
| Total |  | **39** |

* 1. The average number of orders for this 8-day simulation is 39/8 = 4.875 orders per day.

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b) The historical data: 2(.00) + 3(.05) + 4(.25) + 5(.40) + 6(.20) + 7(.08)

+8(.02) +9(.00) = 5.07 orders per day.

c) The two (4.875 and 5.07) are close but because of random variability, not equal. As the length of simulation increases, however, the sample mean should approach the population mean.

# Exercise 4

Soud Suleiman sells insurance on a part-time basis. His records on the number of policies sold per week over a 50 week period are given below as:

# Table 16: Records on the number of policies sold per week

|  |  |
| --- | --- |
| **No. of Policies sold per week** | **Frequency** |
| 0 | 8 |
| 1 | 15 |
| 2 | 17 |
| 3 | 7 |
| 4 | 3 |
| **Total** | 5**0** |

1. Develop a cumulative frequency probability and assign random number intervals corresponding to it
2. Simulate TWO, a 5-day period for Policies sold by Soud Suleiman
3. What is the average number of policies per day for each 5-day simulation period
4. Compare simulated average to the *expected* number of policies based on the two simulation data
5. For each simulation determine the % of weeks during which 2 or more policies are sold.

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**Note:** The random number are obtained from the table for 1st Simulation: 73 41 52 63 and 39 and

2nd simulation: 64 20 81 94 and 25

# SOLUTION

**Table 17: Cumulative frequency probability and assignment of random number intervals**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **No. of Policies** | **Frequency** | **Relative** | **Cumulative** | **Correspondin g** | |
| **sold per week** | **Frequency** | **Frequency** | **Random No.** | |
| 0 | 8 | 0.16 | 0.16 | 0.0 – 16 | |
| 1 | 15 | 0.30 | 0.46 | 1  7 | – 46 |
| 2 | 17 | 0.34 | 0.80 | 4  7 | – 80 |
| 3 | 7 | 0.14 | 0.94 | 8  1 | – 94 |
| 4 | 3 | 0.06 | 1.00 | 9  5 | – 100 |
| **Total** | **50** | **1.00** |  |  | |

# Table 18: Simulation of a 5-day period for Policies

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1st SIMULATION** | | | **2nd SIMULATION** | | |
| **Day** | **Random No.** | **Simulated No. of Policies** | **Day** | **Random No.** | **Simulated No. of Policies** |
| 1 | 73 | 2 | 1 | 63 | 2 |
| 2 | 41 | 1 | 2 | 20 | 1 |
| 3 | 52 | 2 | 3 | 81 | 3 |
| 4 | 63 | 2 | 4 | 94 | 3 |
| 5 | 39 | 1 | 5 | 25 | 1 |
| **Total** |  | **8** | **Total** |  | **10** |

* 1. The average number of policies for 1st simulation of 5-day is 8/5 = 1.6 policies per day and for 2nd simulation of 5-day is 10/5 = 2.0 policies per day
  2. The two simulations (1.6 and 2.0) are close but because of random variability, not equal. As the length of simulation increases, however, the sample mean should approach the population mean.
  3. The % of weeks during which 2 or more policies are sold for 1st simulation is 3/5 x 100 = 60% and 2nd simulation is 3/5 x 100 = 60%

# QUEUING MODEL (THEORY)

* 1. **Fundamental Concepts of Queuing Theory**

# Introduction

The first problem of queuing theory was raised by calls and Erlang was the first who treated congestion problems in the beginning of 20th century. His works inspired engineers,

Mathematicians to deal with queuing problems using probabilistic methods. Queuing theory became a field of applied probability and many of its results have been used in operations research, computer science, telecommunication, traffic engineering, reliability theory, and others. It analyze the shared facility needs to be accessed for service by a large number of jobs or customers. Examples: Waiting lines in cafeterias, hospitals, banks, theaters, airports etc.

# Definition of a queuing Model

A queuing model is a suitable model to represent a service oriented problem where customers arrive randomly to receive some service, the service time being also a random variable.

# Objective of a queuing model

The objective of a queuing model is to find out the optimum service rate and the number of servers so that the average cost of being in queuing system and the cost of service are minimized.

# Application of a queuing model

The queuing models are basically relevant to service oriented organizations and suggest ways and means to improve the efficiency of the service. This model can be applied in the field of business such as banks and booking counters, industries such as servicing of machines, government such as railway or post-office

counters, transportation such as airport and habour and in everyday life such as in

elevators, restaurants, hospitals among others

# Relationship between service and cost

An improvement of service level is always possible by increasing the number of employees. Apart from increasing the cost an immediate consequence of such a step is unutilized or idle time of the servers. In addition, it is unrealistic to assume that a large-scale increase in staff is possible in an organization. Queuing methodology indicates the optimal usage of existing manpower and other resources to improve the service. It can also indicate the cost implication if the existing service facility has to be improved by adding more servers.

# Arrival

The **arrival rate** is the rate at which customers arrive at the service facility during a specified period of time. For example, if 100 customers arrive at a store checkout counter during a 10-hour day, we could say the arrival rate averages 10 customers per hour.

The statistical pattern to the arrival can be indicated through

* + - 1. The probability distribution of the number of arrivals in a specific period of time
      2. The probability distribution of the time between two successive arrival (inter-arrival time)

The number of arrivals is a discrete variable whereas the inter-arrival times are continuous random and variable. A remarkable result in this context is that if the number of arrivals follows a Poisson Distribution, the corresponding inter-arrival time follows An Exponential Distribution. This property is frequently used to derive elegant results on queueing problems

# Service

The time taken by a server to complete service is known as a service time. The service time is a statistical variable and can be studied either as the number of services completed in a given period of time or the time taken to complete the

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service. The data on actual service time should be analyzed to find out the

probability distribution of service time. The number of services completed is a discrete random variable while the service time is a continuous random variable.

# Server

A server is a person or a mechanism through which service of offered. The service may be offered through a single server such as a ticket counter or through several channels such as a train arriving in a station with several platforms. Sometimes the service is to be carried out sequentially through several phrases known as multiphase service. In government, the papers move through a number of phase in terms of official hierarchy till they arrive at the appropriate level where a decision can be taken.

# Time spent in the queuing system

The time spent by a customer in a queuing system is the sum of waiting before service and service time. The waiting time of a customer is the time spent by a customer in a queuing system before the service starts. The probability distribution of waiting time depends upon the probability distribution of inter-arrival time and service time.

# Queue disciple

The queue discipline indicates the order in which members of the queue are selected for service. It is most frequently assumed that the customers are served on the first come first serve basis. T This is commonly referred to as FIFO (First In, First Out) system. Occasionally, a certain group of customers receive priority in service over the others even if they arrive late. This is commonly referred to a Priority Queue. The queue discipline does not always take into account the order of arrival. The server chooses on of the customers to offer service at random. Such a system is known as service in random order (SIRO). While allotting an item with high demand and limited supply such as a test match ticket or share of a public limited company. SIRO system is the only possible way of offering service when it sin not possible to identify the order of arrival.

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The service discipline determines t he rule according to the next customer is

selected. The most commonly used laws are

1. FIFO - First In First Out: who comes earlier leaves earlier
2. LIFO – Last Come First Out: who comes later leaves earlier
3. SIRO – Service In Random Order: the customer is selected randomly
4. PQ –Priority Queue : A category of customer are given precedence to be service earlier

# Calling population

The calling population is the source of the customers to the queuing system, and it can be either *infinite* or *finite.* An infinite calling population assumes such a large number of potential customers that it is always possible for one more customer to arrive to be served. For example, a grocery store, a bank, and a service station are assumed to have infinite calling populations; that is, the whole town or geographic area.

A finite calling population has a specific, countable number of potential customers. It is possible for all the customers to be served or waiting in line at the same time; that is, it may occur that there is not one more customer to be served. Examples of a finite calling population are a repair facility in a shop, where there is a fixed number of machines available to be worked on, a trucking terminal that services a fleet of a specific number of trucks, or a nurse assigned to attend to a specific number of patients.

# Kendall’s Notation

Kendall’s Notation is a system of notation according to which the various characteristics of a queuing model are identified. In 1951 Kendell introduced a set of notations which have become standard in the queuing models. A general queuing system is donated by: (a/b/c) : (d/e) where:

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a = probability distribution of the inter-

arrival time b = probability distribution of the service time c = number of servers in the system

d = maximum number of customers allowed in the system e = queue discipline

# Example 1

(M/M/1) : (∞/FIFO) – indicates a queuing system when the inter-arrival times and service times are exponentially distributed having one server in the system with first in first out discipline and number of customers allowed in the system can be infinite.

# Exercise 1

i. Hence M/M/1 denotes a system with Poisson arrivals, exponentially distributed service times and a single server.

ii .M/G/m denotes an m-server system with Poisson arrivals and generally distributed service times.

1. M/M/r/K/n stands for a system where the customers arrive from a finite- source with n elements where they stay for an exponentially distributed time, the service times are exponentially distributed, the service is carried out according to the request’s arrival by r severs, and the system capacity is K .
2. M/M/1/ / represents a single server that has unlimited queue capacity and infinite calling population, both arrivals and service are Poisson (or random) processes, meaning the statistical distribution of both the inter- arrival times and the service times follow the exponential distribution.
3. M/G/1/ / represents a single server that has unlimited queue capacity and infinite calling population, while the arrival is still Poisson process,

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meaning the statistical distribution of the inter-arrival times still follow the

exponential distribution, the distribution of the service time does not.

# Queuing Model or Queuing Theory Terminology

Queuing theory is the mathematical study of waiting lines (or *queues*) that enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue, and being served by the Service Channels at the front of the queue.

**Traffic Intensity** - The ratio noitazilitu eht ro ytisnetni ciffart eht dellac si µ/ג factor and it determines the degree to which the capacity of service station is utilize

  Mean Rate of Arrival in the Queue (λ) Mean Service Rate (µ)

**Balking** - If a customer decides not to enter the queue since it is too long is called Balking **Reneging** - If a customer enters the queue but after sometimes loses patience and leaves it is called Reneging

**Jockeying** - When there are 2 or more parallel queues and the customers move from one queue to another is called Jockeying

**Waiting Time Cost** - The cost of waiting customers include either the indirect cost of lost business or direct cost of idle equipment and persons.

**Idle Time Cost -** The cost of idle service facilities is the payment to be made to the servers for the period for which they remain idle.

**Transient State of a system -** Queuing analysis involves the system’s behavior over time. If the Operating characteristics vary with time then it is said to be transient state of the system.

**Steady state of a system -** If the behavior becomes independent of its initial conditions (no. of customers in the system) and of the elapsed time is called Steady State condition of the system

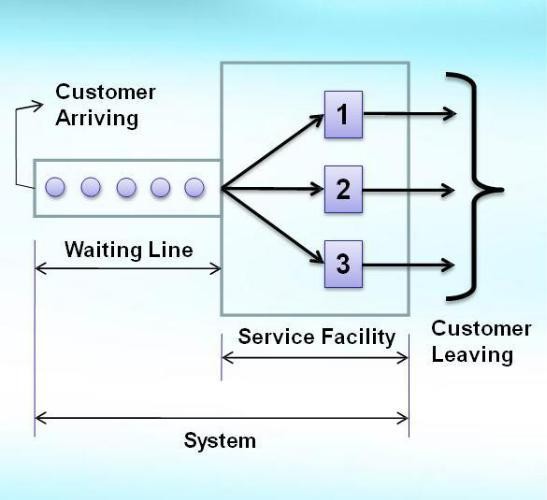
# Queuing Model Application

Queuing Model can be applied to various situations:

1. Where customers are involved such as restaurants, café, super market, airports etc.
2. Very useful in manufacturing units
3. Application for the problem of machine breakdown & repairs
4. Application for the scheduling of jobs in production control
5. Application for the minimization of traffic congestion at tollbooth
6. Provide solution of inventory control problems

# Major Constituents of Queuing System

1. Customer
2. Queue
3. Service Channel



# Figure 2: Major constituents of a queuing system

* + 1. **Assumption in queuing system**

1. The customers arrive for service at a single service facility at random according to Poisson distribution with mean arrival rate ג.

The service time has exponential distribution with mean service rate µ.

1. The service discipline followed is First Come First Served.
2. Customer Behavior is Normal
3. Service facility behavior is Normal
4. The calling source has infinite size
5. The mean arrival rate is less than the mean service rate
6. The waiting space available for customer in the queue is infinite

# Limitations of queuing model

1. The waiting space for the customer is usually limited
2. The arrival rate may be state dependent
3. The arrival process may not be stationary
4. The population of customers may not be infinite and the queuing discipline may not be First Come First Serve
5. Services may not be rendered continuously
6. The Queuing system may not have reached the steady state. It may be, instead, in transient state

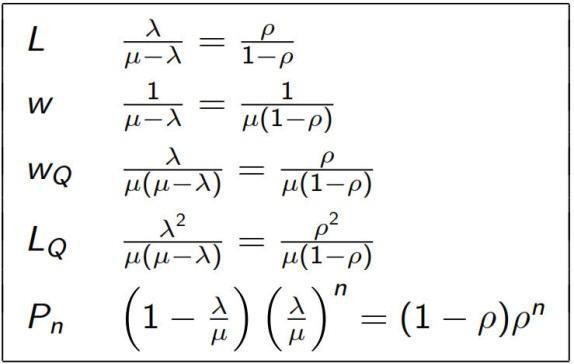
# Queuing Theory Equations

The following Measures of Performance of Queuing Systems are represented as follows:

1. Average number of customers in system (L)
2. Average number of customers in queue (LQ)
3. Average time spent in system per customer (w)
4. Average time spent in queue per customer (wQ)
5. Server utilization (ρ)

# Single-Server Queues (Poisson Arrivals & Infinite Capacity)

**Table 19: Steady-State Parameters of the M /G /1 Queue**



# Single-Server Queues (Poisson Arrivals & Infinite Capacity) Example 1

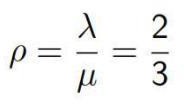
Assume that time between arrivals and service times at a single-chair unisex hair- styling shop have been shown to be exponentially distributed. The values of λ and µ are 2 per hour and 3 per hour, respectively. Computer the following

a ) The server utilization

1. The probabilities of having 0, 1, 2 and 3 or more customers in the shop
2. The number of customers in system, in queue and their waiting times

# Solution

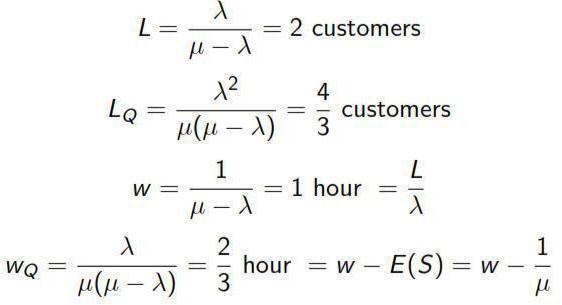
**The server utilization is**



# The probabilities of having 0, 1, 2 and 3 or more customers

−

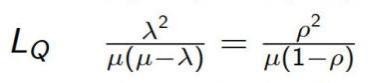
# The number of customers in system, in queue and their waiting times

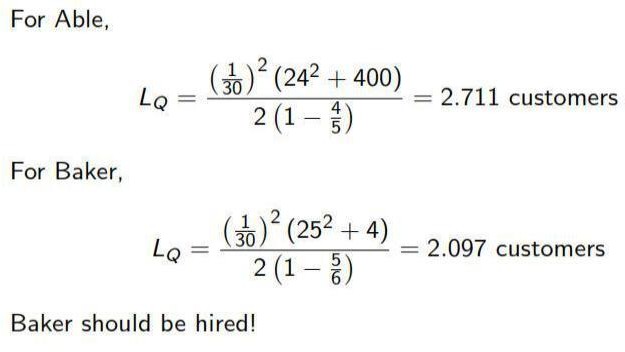


* + 1. **Single-Server Queues (Poisson Arrivals & Infinite Capacity) Example 2**

There are two workers competing for a job; Able and Joy. Able claims an average service time that is faster than Joy’s, but Joy claims to be more consistent, even if not as fast. The arrivals occur according to a Poisson process with a rate of λ = 2 per hour. Able’s statistics are an average service time of 24 minutes with a standard deviation of 20 minutes. Joy’s statistics are an average service time of 25 minutes with a standard deviation of 2 minutes. If the average queue length is the criterion for hiring, which worker should be hired?

# Solution





**Exercise 1**

Kenya Airways has one reservation clerk on duty in its Moi International Airport branch at any given time. The clerk handles information regarding passenger reservation and flight timings. Assume that the number of customers arriving during any given period is passion distribution with and arrival rate 8 per hour and that the clerk can service 1 customer in 6 minutes on an average, with an exponentially distributed service time.

* + - 1. What is the probability that the system is busy?
      2. What is the average time a customer spends in the system?
      3. What is the average length of the queue?
      4. What is the average number of the customers in the system?

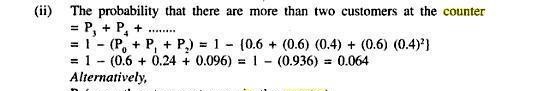
# Solution

**Exercise 2**

A Jua Kali mechanic finds that the time spent on his job has an exponential distribution with an average of 20 minutes. If repairs cars in the order in which they arrive in, and if the arrival is approximately Poisson with an average rate of 8 cars per 8 hours in a day.

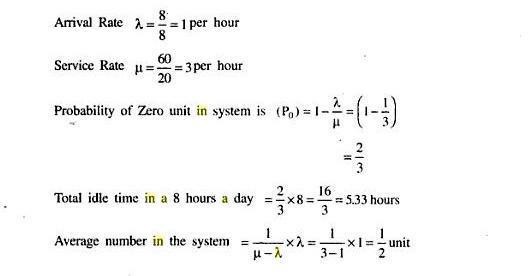
1. What is the mechanic’s expected idle time each day?

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1. How many jobs are on an average in the system?

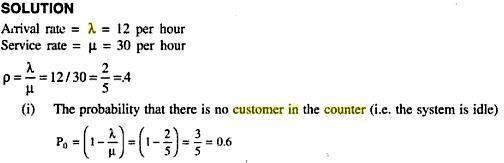
# Solution



**Exercise 3**

Cinema goers arrive at the Nyali Cinemax ticket’s counter at a rate of 12 per hour. There is one clerk serving the customers at a rate of 30 per hour.

1. What is the probability that there are no customers in the counter? (The system is idle)
2. What is the probability that there are more than 2 customers in the counter?
3. What is the probability that there are no customers waiting to be served?
4. What is the probability that a customer is being served and nobody waiting?



# SIMULATION SOFTWARE AND APPLICATIONS

There are many simulation software available such as

1. SIMSCRIPT III
2. GPSS/H
3. ExtendSim
4. Witness
5. Micro Saint Sharp
6. SIMPROCESS
7. ProModel
8. Arena
9. @RISK

They have some features in common. They provide the following features

1. Model development
2. Random variate generation
3. Collection and tabulation of simulation results
4. Time keeping
5. Animation

The following simulation software are the most commonly used in the market

1. SIMPROCESS
2. EXTENDSIM
3. @RISK
4. ARENA

# SIMPROCESS

SIMPROCESS® is a hierarchical modeling tool that combines process mapping, discrete-event simulation, and Activity-Based Costing (ABC) in a single easy to use interface. The SIMPROCESS simulation engine provides the foundation for web-based Decision Support Systems to move Modeling and Simulation to the management desktop.

SIMPROCESS provides ready-made building blocks for constructing dynamic business process models, while the underlying expression language allows experienced programmers to add more complicated business logic. SIMPROCESS is designed for organizations that wish to mitigate the risk associated with implementing dramatic process changes. The tool allows users to quickly and easily analyze various “what-if” scenarios, and by utilizing Java and XML technologies it provides the necessary power and flexibility to meet these organizational needs.

## Features

* Hierarchical Process Mapping
* Object-oriented Modeling
* Activity-Based Costing
* Process Animation
* Standard and Custom Reporting and Graphs
* Standard and Custom Real-Time Plots
* Model Customization through Attributes and Expression Language
* Distribution Library and Distribution Fitting
* File, Spreadsheet, and Database Interfaces
* Import and Export XPDL
* Export to PDF, RTF, PPT, and HTML

# EXTENDSIM

ExtendSim (formerly known as Extend) is a simulation program for modeling discrete event, continuous, agent-based, and discrete rate processes. There are four ExtendSim packages:

* + 1. CP for continuous processes;
    2. OR (operations research) which adds discrete event;
    3. AT (advanced technology) which adds discrete rate, a number of advanced modeling features,
    4. Stat-Fit for statistical distribution fitting; and Suite which adds 3D animation ExtendSim is used for modeling manufacturing, healthcare, supply chain, communications, defense, environmental, agricultural, biological, energy,

reliability, service, information flow, and recreational systems. Sample

applications include resource optimization for food logistics, six sigma process improvements for a hospital emergency department, communication systems, and manufacturing facility design

# @RISK

@RISK (pronounced “at risk”) is software developed by Palisade. It performs risk analysis using Monte Carlo simulation to show you many possible outcomes in your spreadsheet model—and tells you how likely they are to occur. It mathematically and objectively computes and tracks many different possible future scenarios, then tells you the probabilities and risks associated with each different one. This means you can judge which risks to-take and which ones to avoid, allowing for the best decision making under uncertainty.

@RISK also helps you plan the best risk management strategies through the integration of RISKOptimizer, which combines Monte Carlo simulation with the latest solving technology to optimize any spreadsheet with uncertain values. Using genetic algorithms or OptQuest, along with @RISK functions, RISKOptimizer can determine the best allocation of resources, the optimal asset allocation, the most efficient schedule, and much more.

# ARENA

**Arena** is a discrete event simulation and automation software developed by Systems Modeling and acquired by Rockwell Automation in 2000 It uses the SIMAN processor and simulation language. As of June 2014, it is in version 14.7, providing significant enhancements in optimization and animation. It has been suggested that Arena may join other Rockwell software packages under the "FactoryTalk" brand

In Arena, the user builds an experiment *model* by placing *modules* (boxes of different shapes) that represent processes or logic. Connector lines are used to join these modules together and to specify the flow of *entities*. While modules have specific actions relative to entities, flow, and timing, the precise representation of each module and entity relative to real-life objects is subject to the modeler.

Statistical data, such as cycle time and WIP (work in process) levels, can be

recorded and outputted as reports.

Arena can be integrated with Microsoft technologies. It includes Visual Basic for Applications so models can be further automated if specific algorithms are needed. It also supports importing Microsoft Visio flowcharts, as well as reading from or outputting to Excel spreadsheets and Access databases. Hosting ActiveX controls is also supported.

Arena is used by many large companies engaged in simulating business processes.

Some of these firms include

* + 1. General Motors UPS
    2. IBM
    3. Nike
    4. Xerox
    5. Lufthansa
    6. Ford Motor Company and others.

It has been noted that creating a simulation can require more time at the beginning of a project, but quicker installations and product optimizations can reduce overall project time. Arena can simulate diverse operation types, including call centers, for optimizing the use of agents and phone lines, the size and routing of pancake stacks in a food processing facility, and the design of a gold mine.